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BURST FIRING SYSTEMS AND THEIR FIRE CONTROL REQUIREMENTS

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BALLISTIC RESEARCH LABORATORY
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I. INTRODUCTION

Purpose and Scope

This report describes an analysis of aiming techniques which may be applicable to automatic cannons of the 75-mm size range.

The impetus for this work has come from our observation that previous analyses of medium caliber automatic cannons have restricted themselves to analyzing these systems using a single-shot firing technique which we shall henceforth refer to as conventional. Dissemination of these previous analyses has resulted in proposing very sophisticated fire control systems for 75-mm usage.

Whenever one sees something that may be akin to proposing "a sniper scope for a shotgun," one's curiosity is piqued, thus instigating this study.

It should be pointed out that none of the aiming techniques are new, in any sense of the word, to users of automatic cannons, or for that manner single-shot system users. They are new in terms of the analyst's ability to represent their outcomes statistically.

It also is recognized that the user in the field will gravitate to the optimal aiming technique irrespective of our work. However, we hope that showing the potential of automatic cannons to be very effective without the need for sophisticated fire control equipment may increase the chance that the user will be able to demonstrate his prowess with such a system.

We analyzed the following techniques:

- conventional
- spray
- ambush

The nomenclature is ours and is probably unique to our group. However, the terminology provides a good description of the technique. Conventional aiming was described in a previous paragraph. Spray aiming refers to a technique whereby a burst is fired in a preset shot pattern, thus spraying the target. In ambush aiming a curtain of fire is laid down for the target to move through thus ambushing the target.

Background

Before beginning, a review of some basic elements of statistical analysis is in order.

Every firepower system has attributed to it an error budget. This error budget is a mathematical representation of error sources which degrade the system's ability to deliver a projectile (missile, etc.) to the target. These error sources affect the accuracy in different ways and with different frequencies. Some sources have a different value or effect on each round. Muzzle velocity variations are of this type in that the muzzle velocity is different for each round, which obviously affects where that round will hit on the target plane. Error sources of this type are considered as random and determine how "tight" the pattern is at the target plane.

Some error sources remain relatively constant for a given period of time, but can have a different value at another time. These sources are termed variable biases and affect where the midpoint of the pattern will be on the target plane. Cant, crosswind, and range estimation can be considered variable bias errors.

A third category of error sources is referred to as fixed biases. These always have the same value and, therefore, can usually be purged from the system by adjusting the sights.

The number and type of error sources for a given system are numerous, allowing their cumulative effect on accuracy to be represented by Gaussian distributions in vertical and horizontal planes. Thus, the net error budget for systems with no fixed biases is represented by two distributions (one for the vertical and one for the horizontal effects) to describe the variable biases and two distributions for random errors.

II. THE MATHEMATICS OF BURST FIRE

This section describes the general equations used in analyzing various firing techniques of burst fire weapons. It begins with a development of the basic equation in symbolic terms and then substitutes the various statistical functions germane to the problems at hand.

Throughout this section only events which are binary are considered; i.e., an attempt has only two outcomes, total success or total failure. For example, if success is viewed as hitting a target, an attempt results in either success (hit) or failure (miss); no partial credit for "close."

Let $P_{S|\alpha_k}$ represent the probability of success given a condition α_k exists and let $P_{\alpha_k|\beta_j}$ (i) represent the probability that condition α_k will exist on the ith attempt if condition β_j exists. Then the probability of success of the ith attempt, given condition β_j exists, is just

the weighted sum over all values of α_k

$${}^{P}_{S|\beta_{j}}(i) = \sum_{Vk} {}^{P}_{S|\alpha_{k}} * {}^{P}_{\alpha_{k}|\beta_{j}}(i) . \qquad (1)$$

The probability that the ith attempt fails under these conditions is merely

$$\overline{P}_{S|\beta_{j}}(i) = 1 - P_{S|\beta_{j}}(i) . \qquad (2)$$

If n attempts are made, the probability that they all fail, given $\boldsymbol{\beta}_{\,i}$ exists, is

$$P_{F|\beta_{j}}(N) = \overline{P_{S|\beta_{j}}}(1) * \overline{P_{S|\beta_{j}}}(2) * \cdots * \overline{P_{S|\beta_{j}}}(N)$$

$$= \prod_{i=1}^{N} \left[1 - \sum_{V_{k}} P_{S|\alpha_{k}} * P_{\alpha_{k}|\beta_{j}}(i) \right] . \tag{3}$$

To arrive at the unconditional probability of failure, one merely adds all the conditional probabilities $P_{F\,\big|\,\beta\,j}$ (N) weighted by the probability that the condition β_j exists, i.e.,

$$P_{F}(N) = \sum_{Vj} P_{F|\beta_{j}}(N) * P_{\beta_{j}}$$

$$= \sum_{Vj} \left\{ \prod_{i=1}^{N} \left[1 - \sum_{Vk} P_{S|\alpha_{k}} * P_{\alpha_{k}|\beta_{j}}(i) \right] \right\} * P_{\beta_{j}}. \tag{4}$$

Finally, the probability of success is found to be the complement of $\boldsymbol{P}_{_{\boldsymbol{\Sigma}}}(\boldsymbol{N})$, namely

$$P_{S}(N) = 1 - \sum_{V_{j}} \left\{ \prod_{i=1}^{N} \left[1 - \sum_{V_{k}} P_{S \mid \alpha_{k}} * P_{\alpha_{k} \mid \beta_{j}}(i) \right] \right\} * P_{\beta_{j}}.$$
 (5)

For the specific problem at hand, $P_S(N)$ is the probability of killing a target having made n attempts to do so; P_{β} is the probability that a bias, β_j , has influenced the n attempts; $P_{\alpha_k}|_{\beta_j}$ (i) is the probability that the ith attempt succeeds in hitting an area $_{\alpha_k}$ given a bias $_j$; and $_{S|\alpha_k}$ is the probability of killing the target given that area $_k$ was hit.

Let the bias in a firepower system be represented by a density function $\rho_{g}(\nu;\mu)$. The mean of this function, μ , can be viewed as the "fixed bias" of a system whereas the variable ν is merely the "variable bias." Let the impact points be represented by a density function $\rho_{g}(x;\psi_{i}(\nu))$ where x represents the spatial coordinate of the impact point of the system which includes random error and the aimpoint offset dictated for the ith attempt by the specified firing technique. Finally, let $P_{k}(x)$ represent the probability of killing a target given the ordnance impacted point x. Then,

$$\underbrace{P_{K}(N)}_{P_{S}(N)} = 1 - \int_{-\infty}^{\infty} \left\langle \prod_{i=1}^{N} \left\{ 1 - \int_{-\infty}^{\infty} \underbrace{P_{K}(x) \rho_{\alpha}[x; \psi_{i}(v)] dx}_{P_{S}|\alpha_{k}} \right\} \underbrace{\rho_{\beta}(v; \mu) dv}_{P_{\beta}} \right\}. \tag{6}$$

Equation (6) forms the basis for the analysis described where x, ν , and μ are treated as vectors.

Several interesting simplifications can be made of Equation (6) for specific applications. For example, if one is considering munitions which must strike the target to be effective, the range of the integral within the braces reduces to the presented silhouette of the target. If one measures success solely on the basis of hitting the target, one merely sets $P_K(x) = 1$ for all x's contained in the presented silhouette and $P_K(x) = 0$ for all x's elsewhere. Finally, if one does not wish to change aimpoint between attempts, the dependence of this integration on i disappears and Equation (6) reduces to

$$P_{K}(N) = 1 - \int_{-\infty}^{\infty} \left\{ 1 - \int_{-\infty}^{\infty} P_{K}(x) \rho_{\alpha}[x; \psi(v)] dx \right\}^{N} \rho_{\beta}(v; \mu) dv$$
 (7a)

$$= \sum_{j=1}^{N} \frac{N!}{j!(N-j)!} (-1)^{j+1} \int_{-\infty}^{\infty} \left\{ \int_{-\infty}^{\infty} P_{K}(x) \rho_{\alpha}[x;\psi(v)] dx \right\}^{j} \rho_{\beta}(v;\mu) dv (7b)$$

Equation (7b) is a particularly convenient form of the general Equation (6) when the density functions are independent with respect to their variables. The advantages will become apparent in the next section when discussing the conventional aim technique.

III. SEVERAL AIMING TECHNIQUES

Scenario

The objective of this analysis is to find aiming techniques which decreases fire control complexity required to achieve a given level of performance or for a given level of fire control, increase the system's performance. The error budgets employed in this study are on the order of the M60A1E3 fire control. They are listed in Table 1.

TABLE 1. ERROR BUDGETS

	Horizontal	Vertical (σ)
Stationary		
Variable Bias	0.4	0.4
Random Error	0.3	0.3
Moving		
Variable Bias	1.5	0.4
Random Error	0.3	0.3

Two difference scenarios were used in this study. The first has the target moving perpendicular to the gunner's line-of-sight at a crossing velocity of 10 m/s (~ 22 mph). The second has the target moving in a sinusoidal path toward the gunner with a maximum apparent crossing velocity of 10 m/s and a maximum acceleration in the turns of ± 2.5 m/s . In both scenarios there is no accurate lead-angle fire-control computer.

The nonexistence of a lead-angle fire-control computer does not mean that the target is not lead. It does mean that a substantial variation between the "true lead" and the "estimated lead" can exist. This error source is considered as a variable bias since each round has the same error in lead. Without a lead-angle fire-control computer, the variance in "true" and the "estimated" lead can result in a variable bias error distribution with a standard deviation of 1.5 mrad as opposed to one of 0.4 mrad with a perfect (or "true") lead-angle computer. The large variation in horizontal variable bias for a moving target is estimated from a "rule of thumb" prediction that the standard deviation will be roughly 25 percent of the true required lead on a moving target.

Conventional Aiming

Conventional is the term used to define the first aiming technique to be discussed. Under this technique, each round is fired with the same estimated lead on target. This technique is implied when a gunner uses the same hash marks on his sights to fire each round at a moving target. This strategy enables one to use the special cases of the general equation, Equations (7a) or (7b), to perform the analysis.

The objective is to determine the probability of hitting the target; therefore, $P_{\nu}(x) = 1$ over the presented area of the target, i.e., area = [(x,y)]: $x \le x \le x_1$ and $y \le y \le y_1$] and $P_K(x) = 0$ elsewhere. Since both x and y directions are being examined, bivariate normal distributions are chosen as appropriate two-dimensional density functions to describe the situation. It is assumed that the correlation between variables would be very small; thus the correlation coefficient is set equal to zero in these distributions. With this assumption the distribution involving x and y will always be disjoint so that two univariate normal distributions can be used. However, since these distributions involve functions of vusually varying with i which contribute to the integration of the bivariate normal distribution of v, substitution of univariate normal distributions for the bivariate one may not be possible even though there is minimal correlation. The $\psi(v)$ functions, which are usually described by $\psi(v) = v + a$, are defined for this instance as just $\psi(v) = v$. The variable a is set equal to zero since there is no offset. Combining these substitutions yields the following form for Equation (7), namely

$$P_{K}(N) = 1 - \left\{ \int_{-\infty}^{\infty} \left(1 - \left\{ \left[\frac{1}{\sqrt{2\pi}\sigma_{X}} \int_{X_{0}}^{X_{1}} e^{-\frac{1}{2} \left(\frac{x - v_{X}}{\sigma_{X}} \right)^{2}} dx \right] - \frac{1}{2} \left[\frac{1}{\sqrt{2\pi}\sigma_{y}} \int_{y_{0}}^{y_{1}} e^{-\frac{1}{2} \left[\left(\frac{v_{X} - u_{v}}{\sigma_{v_{X}}} \right)^{2} + \left(\frac{v_{Y} - u_{v}}{\sigma_{v_{Y}}} \right)^{2} \right] \right\} \right\} dv_{X} dv_{Y} \right\}$$

$$\left\{ \frac{1}{2\pi\sigma_{v_{X}} \sigma_{v_{Y}}} e^{-\frac{1}{2} \left[\left(\frac{v_{X} - u_{v}}{\sigma_{v_{X}}} \right)^{2} + \left(\frac{v_{Y} - u_{v}}{\sigma_{v_{Y}}} \right)^{2} \right] \right\} dv_{X} dv_{Y} \right\}$$

$$\left\{ \frac{1}{2\pi\sigma_{v_{X}} \sigma_{v_{Y}}} e^{-\frac{1}{2} \left[\left(\frac{v_{X} - u_{v}}{\sigma_{v_{X}}} \right)^{2} + \left(\frac{v_{Y} - u_{v}}{\sigma_{v_{Y}}} \right)^{2} \right] \right\} dv_{X} dv_{Y} \right\}$$

$$\left\{ \frac{1}{2\pi\sigma_{v_{X}} \sigma_{v_{Y}}} e^{-\frac{1}{2} \left[\left(\frac{v_{X} - u_{v}}{\sigma_{v_{X}}} \right)^{2} + \left(\frac{v_{Y} - u_{v}}{\sigma_{v_{Y}}} \right)^{2} \right\} dv_{X} dv_{Y} \right\}$$

$$\left\{ \frac{1}{2\pi\sigma_{v_{X}} \sigma_{v_{Y}}} e^{-\frac{1}{2} \left[\left(\frac{v_{X} - u_{v}}{\sigma_{v_{X}}} \right)^{2} + \left(\frac{v_{Y} - u_{v}}{\sigma_{v_{Y}}} \right)^{2} + \left(\frac{v_{Y} - u_{v}}{\sigma_{v_{Y}}} \right)^{2} \right] dv_{X} dv_{Y} \right\}$$

For this particular technique, the density functions are disjoint with respect to the ν variables since the $\psi(\nu)$ functions are constant over all i's. Therefore, Equation (7b) could also be used to analyze the problem. Its formulation, involving fewer computations, makes it especially convenient when computer time/memory limitations exist. For this case, Equation (7b) with substitutions becomes

$$P_{K}(N) = \sum_{j=1}^{N} \frac{N!}{j! (N-j)!} (-1)^{j+1} \left\{ \int_{-\infty}^{\infty} \left[\frac{1}{\sqrt{2\pi}\sigma_{X}} \int_{X_{0}}^{X_{1}} e^{-\frac{1}{2} \left(\frac{X-\nu_{X}}{\sigma_{X}} \right)^{2}} dx \right]^{j} \cdot \left\{ \int_{-\infty}^{\infty} \left[\frac{1}{\sqrt{2\pi}\sigma_{y}} \int_{Y_{0}}^{Y_{1}} e^{-\frac{1}{2} \left(\frac{Y-\nu_{y}}{\sigma_{y}} \right)^{2}} dy \right]^{j} - \frac{1}{2} \left(\frac{\nu_{y}-\nu_{y}}{\sigma_{y}} \right)^{2} dy \right\} \cdot \left\{ \int_{-\infty}^{\infty} \left[\frac{1}{\sqrt{2\pi}\sigma_{y}} \int_{Y_{0}}^{Y_{1}} e^{-\frac{1}{2} \left(\frac{\nu_{y}-\nu_{y}}{\sigma_{y}} \right)^{2}} dy \right]^{j} \right\} \cdot \left\{ \int_{-\infty}^{\infty} \left[\frac{1}{\sqrt{2\pi}\sigma_{y}} \int_{Y_{0}}^{Y_{1}} e^{-\frac{1}{2} \left(\frac{\nu_{y}-\nu_{y}}{\sigma_{y}} \right)^{2}} dy \right]^{j} \right\} \cdot \left\{ \int_{-\infty}^{\infty} \left[\frac{1}{\sqrt{2\pi}\sigma_{y}} \int_{Y_{0}}^{Y_{1}} e^{-\frac{1}{2} \left(\frac{\nu_{y}-\nu_{y}}{\sigma_{y}} \right)^{2}} dy \right]^{j} \right\} \cdot \left\{ \int_{-\infty}^{\infty} \left[\frac{1}{\sqrt{2\pi}\sigma_{y}} \int_{Y_{0}}^{Y_{1}} e^{-\frac{1}{2} \left(\frac{\nu_{y}-\nu_{y}}{\sigma_{y}} \right)^{2}} dy \right]^{j} \right\} \cdot \left\{ \int_{-\infty}^{\infty} \left[\frac{1}{\sqrt{2\pi}\sigma_{y}} \int_{Y_{0}}^{Y_{1}} e^{-\frac{1}{2} \left(\frac{\nu_{y}-\nu_{y}}{\sigma_{y}} \right)^{2}} dy \right]^{j} \right\} \cdot \left\{ \int_{-\infty}^{\infty} \left[\frac{1}{\sqrt{2\pi}\sigma_{y}} \int_{Y_{0}}^{Y_{1}} e^{-\frac{1}{2} \left(\frac{\nu_{y}-\nu_{y}}{\sigma_{y}} \right)^{2}} dy \right]^{j} \right\} \cdot \left\{ \int_{-\infty}^{\infty} \left[\frac{1}{\sqrt{2\pi}\sigma_{y}} \int_{Y_{0}}^{Y_{1}} e^{-\frac{1}{2} \left(\frac{\nu_{y}-\nu_{y}}{\sigma_{y}} \right)^{2}} dy \right]^{j} \right\} \cdot \left\{ \int_{-\infty}^{\infty} \left[\frac{1}{\sqrt{2\pi}\sigma_{y}} \int_{Y_{0}}^{Y_{1}} e^{-\frac{1}{2} \left(\frac{\nu_{y}-\nu_{y}}{\sigma_{y}} \right)^{2}} dy \right]^{j} \right\} \cdot \left\{ \int_{-\infty}^{\infty} \left[\frac{1}{\sqrt{2\pi}\sigma_{y}} \int_{Y_{0}}^{Y_{1}} e^{-\frac{1}{2} \left(\frac{\nu_{y}-\nu_{y}}{\sigma_{y}} \right)^{2}} dy \right]^{j} \right\} \cdot \left\{ \int_{-\infty}^{\infty} \left[\frac{1}{\sqrt{2\pi}\sigma_{y}} \int_{Y_{0}}^{Y_{1}} e^{-\frac{1}{2} \left(\frac{\nu_{y}-\nu_{y}}{\sigma_{y}} \right)^{2}} dy \right]^{j} \right\} \cdot \left\{ \int_{-\infty}^{\infty} \left[\frac{1}{\sqrt{2\pi}\sigma_{y}} \int_{Y_{0}}^{Y_{1}} e^{-\frac{1}{2} \left(\frac{\nu_{y}-\nu_{y}}{\sigma_{y}} \right)^{2}} dy \right]^{j} \right\} \cdot \left\{ \int_{-\infty}^{\infty} \left[\frac{1}{\sqrt{2\pi}\sigma_{y}} \int_{Y_{0}}^{Y_{1}} e^{-\frac{1}{2} \left(\frac{\nu_{y}-\nu_{y}}{\sigma_{y}} \right)^{2}} dy \right] \right\} \cdot \left\{ \int_{-\infty}^{\infty} \left[\frac{1}{\sqrt{2\pi}\sigma_{y}} \int_{Y_{0}}^{Y_{1}} dy \right] dy \right\} \cdot \left\{ \int_{-\infty}^{\infty} \left[\frac{1}{\sqrt{2\pi}\sigma_{y}} \int_{Y_{0}}^{Y_{1}} dy \right] dy \right\} \cdot \left\{ \int_{-\infty}^{\infty} \left[\frac{1}{\sqrt{2\pi}\sigma_{y}} \int_{Y_{0}}^{Y_{1}} dy \right] dy \right\} \cdot \left\{ \int_{-\infty}^{\infty} \left[\frac{1}{\sqrt{2\pi}\sigma_{y}} \int_{Y_{0}}^{Y_{1}} dy \right] dy \right\} \cdot \left\{ \int_{-\infty}^{\infty} \left[\frac{1}{\sqrt{2\pi}\sigma_{y}} \int_{Y_{0}}^{Y_{1}} dy \right] dy \right\} \cdot \left\{ \int_{-\infty}^{\infty} \left[\frac{1}{\sqrt{2\pi}\sigma_{y}} \int_{Y_{0}}^{Y_{1}} dy \right] dy \right\} \right$$

After mechanizing Equation (8), the results for the first scenario ake the following form. Figure 1 and Table 2 show the probability of hitting a moving target at least once given that a single shot is fired (lower curve), given that three rounds are fired (middle curve), and given that five rounds are fired (upper curve).

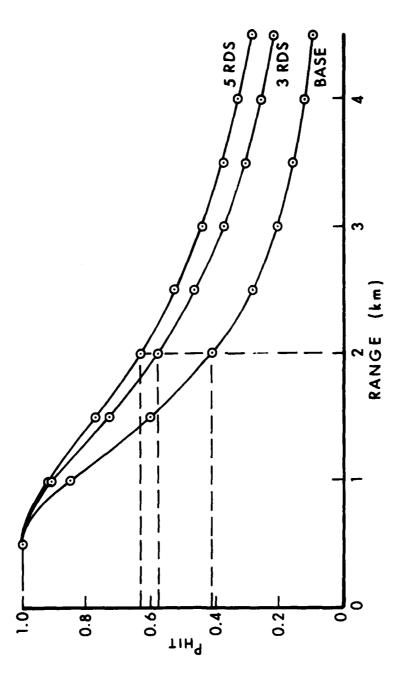


Figure 1. P_{HIT} for Conventional Aiming

TABLE 2. PHIT FOR CONVENTIONAL AIMING GIVEN A CROSSING PATH

Range (m)	1-Round Baseline	3-Round Burst	5-Round Burst
500	1.00	1.00	1.00
1000	0.85	0.91	0.92
1500	0.60	0.73	0.77
2000	0.41	0.58	0.63
2500	0.29	0.46	0.52
3000	0.21	0.37	0.44
3500	0.16	0.31	0.37
4000	0.13	0.26	0.32
4500	0.10	0.22	0.28
5000	0.08	0.19	0.25

At 2000 m, the probability of hitting the target with a single round is on the average 0.41. When firing three rounds with the same lead estimation for each round, the probability of hitting the target at least once is 0.58 and with five rounds under the same conditions it rises to 0.63.

One of the first things that strikes the observer is that the hit probability for three rounds as compared to one is not so great as one would intuitively guess. This is particularly evident in the small gain observed between five and three rounds. To visualize the phenomenon which precipitates this curiosity, picture a system which has no random error; that is, each round flies to the same point as the previous for a given time duration. Then either all rounds will hit the target (if the combination of variable- and fixed-bias errors remain on the target) or all rounds will miss the target (if the combination error is off the target).

Thus the probability of a three- or five-round burst resulting in at least one hit is equal to (in this extreme example) the probability that a single round hits the target. In other words, if the system has no random error, all three curves would be superimposed on each other. What separates them is the dispersion about the aimpoints, i.e., random error.

In any case this conventional aiming technique for a crossing target has been analyzed and the curves in Figure 1 are based on the "natural" dispersions of the rounds in three- and five-round bursts.

One has to look closely at the peculiarities of the sinusoidal path when examining the results from the second scenario. At some points in the path the target resembles a crossing target from the first scenario. At other times it is apparent that some turning maneuver is being performed. Accurate lead prediction by the gunner is thus very difficult. This can better be understood by two examples of such a maneuver. Picture a target executing a sinusoidal path in an open field. Unless a gunner has been observing the target for some time it would be quite difficult to differentiate it from a regular crossing target. The gunner would probably lead the target as if it were crossing while the target has turned in the meantime. However, if the target were proceeding along a road having the same sinusoidal path, the gunner would probably lead the target correctly knowing that it would turn. To accommodate both situations, the sinusoidal path had to be sampled at various times to gather information on the straight and turning portions of the path. Different probabilities of hit were obtained at each sampling. Therefore, all results for the sinusoidal path will list a minimum and maximum value of $P_{\rm HIT}$ reflecting bad and good lead predictions.

Table 3 lists the results for conventional aiming given a sinusoidal path. The values for the five- and three-round bursts exhibit the same trend as in scenario one; i.e., the probabilities of hit are slightly higher for five-round bursts as compared to three-round bursts which are in turn higher than the one-round baseline case. In comparison to the first scenario, the maximum probabilities of hit show a slight degradation except for the five-round burst which is slightly higher.

TABLE 3. PHIT FOR CONVENTIONAL AIMING GIVEN A SINUSOIDAL PATH

		ound line	3-Roui Bursi		5-Rou Burs	
	Min.	Max.	Min.	Max.	Min.	Max.
500	0.97	1.00	0.98	1.00	1.00	1.00
1000	0.60	0.85	0.71	0.90	0.84	0.93
1500	0.27	0.59	0.48	0.72	0.63	0.82
2000	0.17	0.39	0.30	0.55	0.47	0.73
2500	0.09	0.25	0.18	0.44	0.34	0.65
3000	0.04	0.15	0.10	0.36	0.21	0.54
3500	0.02	0.09	0.05	0.29	0.11	0.41
4000	0.01	0.05	0.03	0.22	0.06	0.29
4500	0.00	0.04	0.02	0.16	0.03	0.20
5000	0.00	0.03	0.01	0.12	0.01	0.14

An analysis restricted to this aiming technique quickly concludes that as the first round hit probability goes, so goes (albeit higher) a three- or five-round burst. Since high first round hit probabilities usually require complex, expensive fire control even the bursts would require complex, expensive fire control. Thus, conventional aiming is somewhat unattractive.

Spray Aiming

Spray aiming is the descriptive term used to define a technique by which the rounds in a burst are laid out in a preset pattern about the aimpoint. This pattern can be defined by choosing certain offsets in either the x- or y-direction or both and plugging them into the $\psi_i(\nu)$ functions of Equation (6). These functions are defined by the linear combinations $\psi_i(\nu) = \nu + a_i$. To simplify matters in this particular analysis, the vertical offsets, designated b_i , are set equal to zero. Meanwhile the horizontal offsets are defined such that $a_{i+1} = a_i + \Delta$ where the middle round of the burst exhibits no offset.

The objective, again, is to hit the target, which means that $P_K(x) = 1$ over the presented silhouette of the area of the target and $P_K(x) = 0$ elsewhere. Bivariate normal distributions are deemed appropriate descriptions of the scenario. Minimal (zero) correlation is assumed for both functions which allows univariate normal distributions to be substituted for the bivariate of x and y. With these insertions the basic equation for both spray and curtain aiming is

$$P_{K}(N) = 1 - \left(\int_{-\infty}^{\infty} \left\{ \left[\frac{N}{i} \left[1 - \sqrt{\frac{1}{\sqrt{2\pi}\sigma_{X}}} \int_{X_{0}}^{X_{1}} e^{-\frac{1}{2} \left(\frac{X - v_{X} - a_{1}}{\sigma_{X}} \right)^{2}} dx \right] \right\} \right\}$$

$$\left[\frac{1}{\sqrt{2\pi}\sigma_{y}} \int_{y_{0}}^{y_{1}} e^{-\frac{1}{2} \left(\frac{y - v_{y} - b_{1}}{\sigma_{y}} \right)^{2}} dy \right] \left\{ \int_{2\pi\sigma_{v_{X}}}^{2\pi\sigma_{v_{X}}} e^{-\frac{1}{2} \left(\frac{v_{X} - u_{v_{X}}}{\sigma_{v_{X}}} \right)^{2}} + \left(\frac{v_{y} - u_{v_{y}}}{\sigma_{v_{y}}} \right)^{2} \right] \right\} dv_{X} dv_{y} \right\}.$$

$$(9)$$

Table 4 and Figure 2 show the hit probability resulting from this technique when the optimal spacing between points in the spray is achieved against a crossing target. The single-shot hit probability is included as a benchmark. Note that the hit probabilities of the burst have significantly increased and the difference between a three- and five-round burst has become more evident. The hit probabilities at 2000 m are now 0.41, 0.74, and 0.84 for one, three, and five rounds, respectively, as compared to 0.41, 0.58, and 0.63 for conventional aiming.

TABLE 4. PHIT FOR SPRAY AIMING GIVEN A CROSSING PATH

Range (m)	1-Round Baseline	3-Round Burst	5-Round Burst
500	1.00	1.00	1.00
1000	0.85	0.99	1.00
1500	0.60	0,89	0.95
2000	0.41	0.74	0.84
2500	0.29	0.59	0.72
3000	0.21	0.47	0.60
3500	0.16	0.38	0.50
4000	, 0.13	0.31	0.42
4500	0.10	0.25	0.36
5000	0.08	0.21	0.31

Table 5 and Figure 3 merely allow comparison between a single-shot system with a stationary target's biases and the burst with a moving target's biases. What is being compared is a 0.4 by 0.4 mrad system with a 1.5 by 0.4 mrad system. Note that the three-round burst option is equivalent to a single round. Of course, it cost two more rounds, but then again, nothing is free.

Table 6 gives the minimum and maximum probabilities of hit for the second scenario using the spray aiming technique with a spray angle of 0.25 mrad. A constant spray angle was chosen instead of an optimal spray angle, because enough fluctuations exist in the results due to the path without compounding or confounding them. The probabilities exhibit minimal increases as compared to the conventional aiming technique. This is contrary to the results obtained with a crossing path. The results obtained for the first scenario suggest a method which may significantly reduce the fire-control requirements of automatic antiarmor cannons or, conversely for a given fire control, may significantly increase the effectiveness. However, for targets traveling a sinusoid path, conventional aiming yields equivalent results.

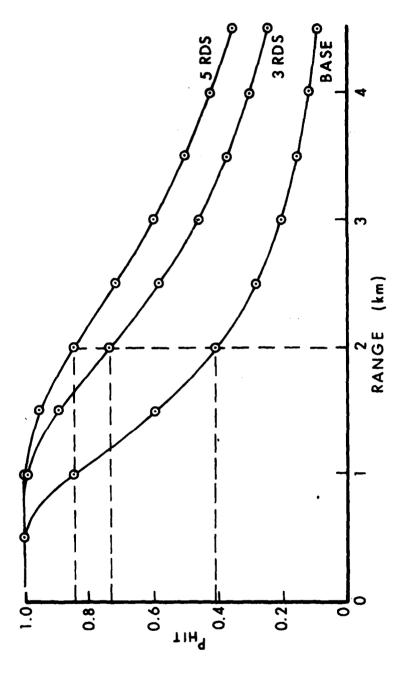


Figure 2. P_{HIT} for Spray Aiming

TABLE S. POTENTIAL FIRE CONTROL TRADEOFFS

	Single Round w/	Spray	Aiming
Range (m)	Perfect Lead Angle Computer	3-Round Burst	5-Round Burst
500	1.00	1.00	1.00
1000	0.98	0.99	1.00
1500	0.87	0,89	0.95
2000	0.73	0.74	0.84
2500	0.60	0.59	0.72
3000	0.49	0.47	0.60
3500	0.40	0.38	0.50
4000	0.33	0.31	0.42
4500	0.27	0.25	0.36
5000	0.23	0,21	0.31

TABLE 6. $P_{\mbox{\scriptsize HIT}}$ FOR SPRAY AIMING GIVEN A SINUSOIDAL PATH

	1-Ro Basel		_	ound rst		ound rst
	Min.	Max.	Min.	Max.	Min.	Max.
500	0.98	1,00	0.98	1.00	1.00	1.00
1000	0.61	0.85	0.74	0.90	0.83	0.92
1500	0.28	0.59	0.49	0.72	0.60	0.80
2000	0.18	0.38	0.33	0.54	0.44	0.65
2500	0.10	0.25	0.20	0.42	0.30	0.59
3000	0.05	0.15	0.11	0.34	0.22	0.53
3500	0.02	0.08	0.06	0.29	0.13	0.43
4000	0.01	0.06	0.03	0.23	0.06	0.32
4500	0.00	0.04	0.02	0,17	0.03	0.24
5000	0.00	0.03	0.01	0.13	0.02	0.16

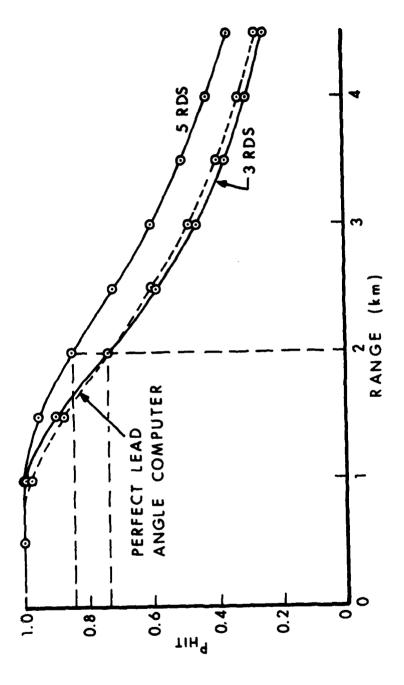


Figure 3. Potential for Fire Control Tradeoffs

Ambush Aiming

Ambush aiming is the name given to the last technique under analysis. In this technique, the gunner aims at a single point in front of the target and lays down a curtain of fire through which the target will pass, thereby ambushing the target.

Equation (9) is employed to analyze the situation. The differences occur in the $\psi_i(\nu)$ functions. They still have the form $\psi_i(\nu) = \nu + a_i$; however, the a_i 's are described in another way. The a_1 term represents the lead-angle slew of the gun for the first round which is approximately the velocity estimation of the target multiplied by the time of flight of the projectile. The horizontal offsets for the subsequent rounds of the burst are defined as follows: $a_{i+1} = a_i + v/rof$ where v is the velocity of the target and rof is the rate of fire of the gun. Since it was not anticipated that the target would move in a vertical direction, these offsets, b_i , are all set equal to zero.

Figures 4 and 5 and Tables 7 and 8 present the hit probabilities of bursts fired at the rate of two and four rounds per second, respectively. Obviously, rate of fire strongly affects the worth of this technique since the faster you fire the less distance the target has traversed. Even so, at 2000 m with a rate of fire of two rounds per second, the probabilities of hit obtained for the one-, three-, and five-round cases of 0.41, 0.66, and 0.67, are still greater than those obtained for conventional aiming, 0.41, 0.58, and 0.63. The most interesting trend can be seen in Figure 6 and Table 9 which show that the hit probability of a three-round burst is nearly constant if one positions the curtain between 6 and 12 mrad in front of the target, i.e., allowing a 6-mrad error in velocity estimation. This is due to the distance traveled by the target while the three-rounds are in flight and impacting. These margins of error are dependent on target velocity and rate of fire, but the results are encouraging.

For the sinusoidal path with a rate of fire of two rounds per second, the probabilities of hit for the best ambush position were used. Again the path was sampled at various times yielding minimum and maximum probabilities of hit shown in Table 10. The ambush technique gives better results than conventional aiming for the three- and five-round bursts. The results are comparable with those of the first scenario.

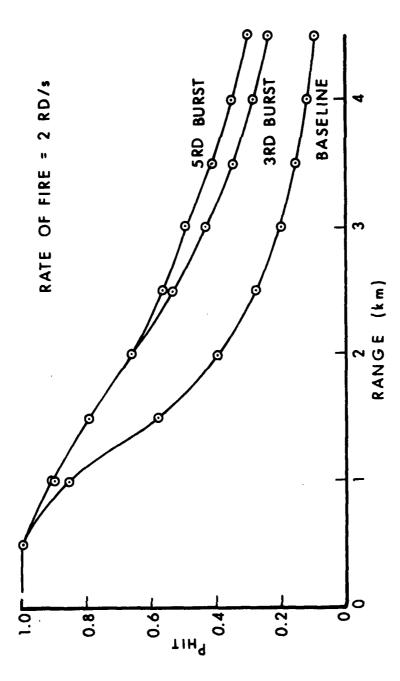


Figure 4. $P_{
m HIT}$ for Ambush Aiming at Two Rounds Per Second

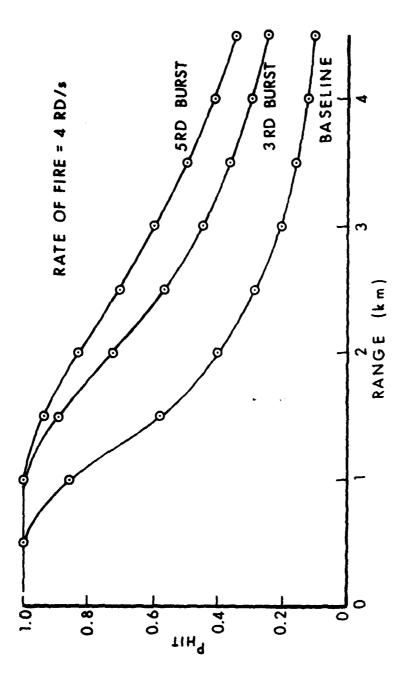


Figure 5. $P_{\rm HIT}$ for Ambush Aiming at Four Rounds Per Second

TABLE 7. $P_{\mbox{\scriptsize HIT}}$ FOR AMBUSH AIMING AT TWO ROUNDS PER SECOND GIVEN A CROSSING PATH

Range (m)	1-Round Burst	3-Round Burst	5-Round Burst
500	1.00	1.00	1.00
1000	0.85	0.90	0.90
1500	0.60	0.79	0.79
2000	0.41	0.66	0.67
2500	0.29	0.54	0.57
3000	0.21	0.44	0.49
3500	0.16	0.36	0.42
4000	0.13	0.30	0.36
4500	0.10	0.25	0.32
5000	0.08	0.21	0.28

TABLE 8. $P_{\mbox{\scriptsize HIT}}$ FOR AMBUSH AIMING AT FOUR ROUNDS PER SECOND GIVEN A CROSSING PATH

Range (m)	1-Round Burst	3-Round Burst	5-Round Burst
500	1.00	1.00	1.00
1000	0.85	0.99	0.99
1500	0.60	0.89	0.94
2000	0.41	0.73	0.83
2500	0.29	0.57	0.72
3000	0.21	0.46	0.60
3500	0.16	0.37	0.50
4000	0.13	0.30	0.42
4500	0.10	0.25	0.36
5000	0.08	0.21	0.31

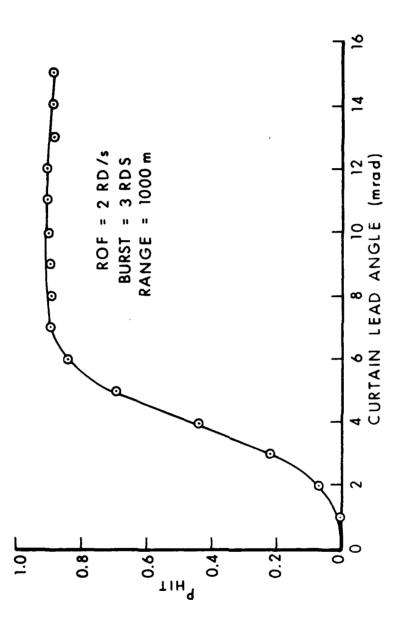


Figure 6. $P_{\mbox{\scriptsize HIT}}$ for Ambush Aiming at 1000 M

TABLE 9. $P_{\mbox{HIT}}$ FOR AMBUSH AIMING AT 1000 M GIVEN A CROSSING PATH

Lead Angle			2 rd/s irst	- -	4 rd/s
(mrad)	Base		5	3	5
1	0.02	0.02	0.02	0.02	0.02
2	0.08	0.08	0.08	0.08	0.08
3	0.22	0.22	0.22	0.22	0.22
4	0.45	0.45	0.45	0.45	0.45
5	0.69	0.69	0.69	0.70	0.70
6	0.83	0,85	0.85	0.87	0.87
7	0.82	0.90	0.90	0.96	0.96
8	0.67	0.88	0.88	0.98	0.98
9	0.43	0.87	0.87	0.99	0.99
10	0.20	0.89	0.89	0.98	0.99
11	0.07	0.92	0.92	0.95	0.99
12	0.02	0.91	0.91	0.86	0.99
13	0.00	0.88	0.88	0.68	0.99
14	0.00	0.87	0.87	0.43	0.99
15	0.00	0.88	0.88	0.20	0.98

Table 10. $\boldsymbol{P}_{\mbox{\scriptsize HIT}}$ for ambush aiming given a sinusoidal path

	Base	l-Round Baseline		3-Round Burst		5-Round Burst	
	Min.	Max.	Min.	Max.	Min.	Max.	
500	0.90	0.99	1,00	1.00	1.00	1.00	
1000	0.63	0.80	0.80	0.99	0.86	0.99	
1500	0.41	0.54	0.52	0.87	0.75	0.93	
2000	0.25	0.37	0.41	0.69	0.66	0.80	
2500	0.16	0.26	0.36	0.53	0.47	0.68	
3000	0.14	0.19	0.28	0.42	0.35	0.56	
3500	0.11	0.14	0.21	0.34	0.30	0.46	
4000	0.07	0.11	0.17	0.27	0.28	0.38	
4500	0.06	0.09	0.16	0.22	0.22	0.32	
5000	0.05	0.07	0.13	0.19	0.18	0.28	

SUMMARY

This report presented the results of our analysis of aiming techniques which could be used by the 75-mm system.

The results are encouraging, especially for simple crossing targets. Whereas previous studies have concluded that an automatic cannon requires a sophisticated fire control, there is now evidence that a relatively simple system may suffice and still have a superior capability. For targets maneuvering along sinusoidal paths the results are not as encouraging, but considering that most first-order lead-angle fire-control computers would not account for such a path, these techniques could be useful.

We conclude that it is no longer obvious that automatic cannons of the 75-mm type require sophisticated fire control equipment to be effective.

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